

Thermal Problems of the Injection Laser

Abstract: Heat is produced during the operation of an injection laser. The thermal conduction problems associated with the flow of the heat away from the junction region have been solved and the temperature increase of the junction has been calculated for several simple model cases. The results have been applied to the calculation of thermal limitations on the performance of gallium arsenide lasers.

Introduction

Certain applications of lasers require operation at high power levels. One limitation on the power level at which a solid state laser may be operated is thermal. Generally, not all of the pumping power delivered to the laser is emitted as light; a large fraction of it is converted into heat. The heat raises the temperature of the laser. If the temperature rise is sufficiently great the character of the electronic processes in the crystal will be altered to the extent that satisfactory laser operation is not possible. The present paper attempts to estimate the limitations which these thermal effects place on the capabilities of p - n junction lasers.

The obvious way to avoid the problem of heating of the junction laser is to place the junction in contact with a thermal bath. The practicality of this solution is, however, limited by the difficulty of transferring heat to, for example, a liquid bath. To illustrate, consider the case of a GaAs junction laser operated at 77°K, the temperature of boiling liquid nitrogen. The rate at which heat is produced at the junction is approximately equal to the threshold current density, about 3000 ampere/cm² for a typical laser, times the energy gap, 1.5 volts for GaAs, giving a product of the order of 4500 W/cm². The maximum rate at which heat can be transferred to liquid nitrogen by nucleate boiling is, however, only 10 W/cm². The laser is cooled by attaching the junction to a large heat sink that reduces the power density at which heat is transferred to the liquid nitrogen by a factor of at least several hundred and, preferably, of several thousand. The dimensions of the heat sink must be orders of magnitude larger than the dimensions of the junction. The thermal problem involved is, then, essentially that of the flow of heat from the junction into a large body of solid material which constitutes the heat sink. The

temperature of the heat sink at very large distances from the junction is equal to the temperature of the thermal bath. This kind of resistance, that which is encountered by the flow of a current from a bounded contact area into a semi-infinite medium, is known as *spreading resistance*.

This paper presents solutions to certain idealized heat flow problems which are intended to approximate the flow of heat from a junction into an adjoining body of solid material. The problems are idealized in order to make them soluble by considering simplified geometrical shapes and by assuming that the material parameters (the thermal conductivity and the specific heat) in the equation of heat conduction are independent of temperature. These assumptions are necessary to preserve the homogeneous nature of the equation of heat conduction.

Part 1 presents solutions to time dependent heat flow problems which relate to pulsed operation of the injection laser. Part 2 describes the solution of the steady state problems of heat conduction which pertain to continuous operation of the laser. The first problem has been treated by Smith and Lasher¹ and the second by Mayburg², both of whom, however, adopted approximations and points of view somewhat different from those presented here. In Part 3, the results of Parts 1 and 2 are applied to gallium arsenide lasers. Certain mathematical details are presented in Appendices A and B, and a resume of notation is given in Appendix C.

In the model injection lasers to be treated, heat is produced at a junction plane S at a rate jV per unit area, where j is the electrical current density and V is the voltage drop in the junction, very nearly the gap voltage. Heat may also be produced by the joule or resistive heating caused by the current in its passage through the material

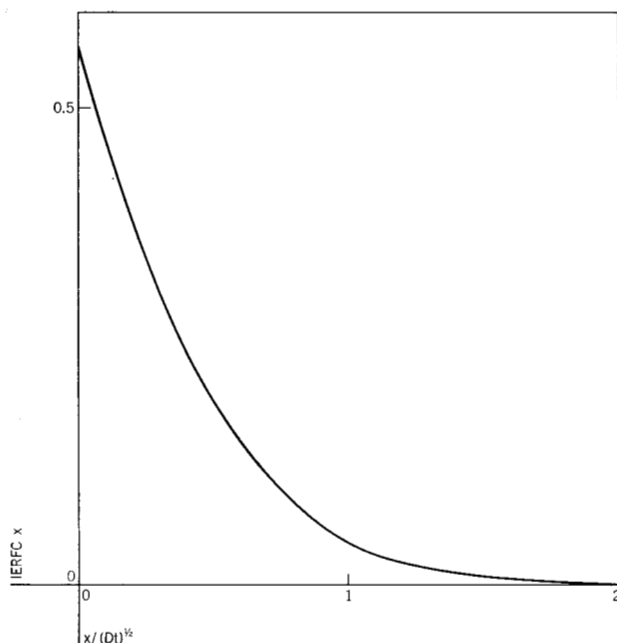


Figure 1 The form of the dependence of temperature on distance from the source during heat flow into a one dimensional heat sink. Note that little thermal energy has penetrated beyond $x = (Dt)^{1/2}$.

surrounding the junction. The model is pessimistic (from the point of view of high-power laser operation) in neglecting the loss of energy as light.

The model is characterized by certain parameters as follows: j_t is the threshold current density. No lasing takes place if the current density j is less than j_t . If j is greater than j_t , the current density $(j - j_t)$ contributes to stimulated emission. Now, j_t depends on the temperature at the junction in a form which, to an adequate degree of approximation, can be written

$$j_t = j_0 \exp(\Delta T/T_1). \quad (1)$$

Here j_0 is the threshold current at the bath temperature T which the laser assumes in the absence of heat sources, $T + \Delta T$ is the temperature of the junction, and T_1 is a characteristic parameter of the laser material.

The thermal conductivity of the material which carries heat from the junction is denoted by κ . The material may be the laser material itself or it may be some metal to which the laser has been attached. In the former case κ is temperature dependent, frequently being proportional to the reciprocal of the temperature, but in the latter case it is essentially temperature independent. Here the latter case will be assumed as a realistic one which has the advantage of avoiding the analytical complexities associated with the treatment of a temperature dependent κ .

κ appears in the equation of heat conduction in its ratio to the specific heat per unit volume C . The quantity $D \equiv \kappa/C$ has the dimensions of a diffusion constant and is called the thermal diffusivity. C is strongly temperature dependent in certain temperature regions. Our treatment is approximate in that C is assumed to be a constant for a given laser and bath temperature.

The last material parameter that characterizes the laser is the electrical conductivity, σ , of the material which carries current to the laser junction. Although σ is structure sensitive it is about 10^3 (Ohm-cm) $^{-1}$ for many heavily doped semiconductors, and is nearly independent of temperature. Neglect of the temperature dependence of σ is a quite reasonable approximation. In metals σ and κ are related by the Wiedemann-Franz Law,

$$\frac{\kappa}{\sigma} = \frac{\pi^2}{3} \left(\frac{k}{q} \right)^2 T, \quad (2)$$

but in semiconductors (κ/σ) is much greater than the value given by Eq. (2).

1. Pulsed operation of the injection laser

Consider first a one-dimensional case in which heat is being produced at a rate jV on a plane, the yz plane, say, and flows away into a solid extending to infinity in the $+x$ direction. The plane of heat production represents the junction plane.

Suppose that the laser is initially at a uniform temperature T with no current flowing. The current is turned on at time $t = 0$. The temperature of the laser then begins to rise, and at a point x and time t is $T + \Delta T(x, t)$ with

$$\Delta T(x, t) = (jV/\kappa)(4Dt)^{1/2} \text{ierfc}[x/(4Dt)^{1/2}]. \quad (3)$$

The function ierfc has been tabulated³ and is shown in Fig. 1. The point illustrated by Fig. 1 is that practically none of the heat energy has penetrated beyond $x = 2(Dt)^{1/2}$. The semi-infinite approximation is good if the extent of the medium in the $+x$ direction is many times $(Dt)^{1/2}$.

At the junction $x = 0$, the temperature rise is

$$\Delta T = (jV/\kappa)(4Dt/\pi)^{1/2}. \quad (4)$$

The temperature of the junction increases in accord with Eq. (4) until the current is turned off at some time t_1 . The threshold current increases according to Eq. (1). If the current is not turned off before the threshold current density reaches the actual current density j , the production of stimulated emission will cease at another time t_0 when this occurs.

We now investigate the question: What is the maximum amount of stimulated light energy that can be produced by an electrical pulse? The amount of stimulated light energy per unit area is proportional, in the present model,

to the integral of the excess of current over threshold current

$$W = V \int_0^{t_1} (j - j_t) dt, \quad (5)$$

it being assumed that $t_1 \leq t_0$. The factor of proportionality is the efficiency, a dimensionless number less than unity. The time dependence of j_t in Eq. (5) is obtained from Eqs. (1) and (4) and is

$$j_t = j_0 \exp \frac{jV}{\kappa T_1} \left(\frac{4Dt}{\pi} \right)^{1/2}. \quad (6)$$

It is advantageous to normalize current densities to the material parameter j_0 . Thus we define

$$j_t^* = j_t / j_0, \quad (7)$$

and

$$j^* = j / j_0. \quad (8)$$

The remaining material parameters then appear in the combination

$$\frac{\pi}{4D} \left(\frac{\kappa T_1}{j_0 V} \right)^2 = \frac{\pi \kappa C T_1^2}{4 j_0^2 V^2} \equiv t_N, \quad (9)$$

which has the dimensions of time. Thus it is also convenient to normalize the time scale to t_N by defining $t^* = t/t_N$. Then Eq. (5) can be rewritten

$$W = j_0 V t_N \int_0^{t_1^*} [j^* - \exp(j^* \sqrt{t^*})] dt^*. \quad (10)$$

An example of the time variation of the integrand in Eq. (10) is shown in Fig. 2.

We can now find that value of j^* which maximizes W for a given pulse length, t_1^* . The solution of this maximization problem is described in Appendix A and presented in Fig. 3, in which the value of j^* that maximizes W and the resulting value of W (referred to $j_0 V t_N$) are plotted as functions of the pulse length, t_1^* . The larger the pulse length, the smaller the j^* and the greater the energy, up to a point. When, however, $t_1^* = 0.0394$, then $t_1^* = t_0^*$; that is, the threshold current has reached the maximizing current itself at the end of the pulse, and lasing stops even if the current is not turned off. Then $W = 0.255 j_0 V t_N$ and $j^* = 13.6$. This represents an absolute maximum limit on the energy of a single pulse. It is not possible to increase W by further lengthening the pulse. To be very specific, the maximum pulse energy above threshold which can be obtained per unit area according to the present model is

$$W_{\max} = (0.255) \frac{\pi \kappa C T_1^2}{j_0 V}. \quad (11)$$

The stimulated light output is less than $W_{\max}$ by an

efficiency factor, the fraction of the electrical energy above threshold which appears as stimulated light.

Note in Fig. 3 the relative insensitivity of the total energy to the pulse duration. Thus, for example, if it is required that t_1 should be only one-tenth of t_0 , the value corresponding to maximum energy, W is still 70% of its value at the maximum. The current density required to

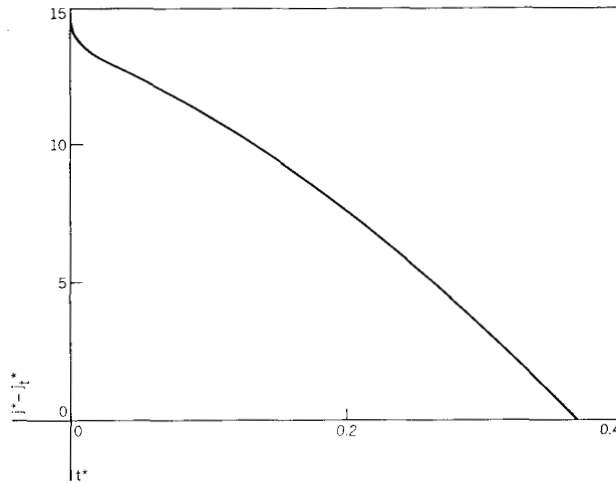
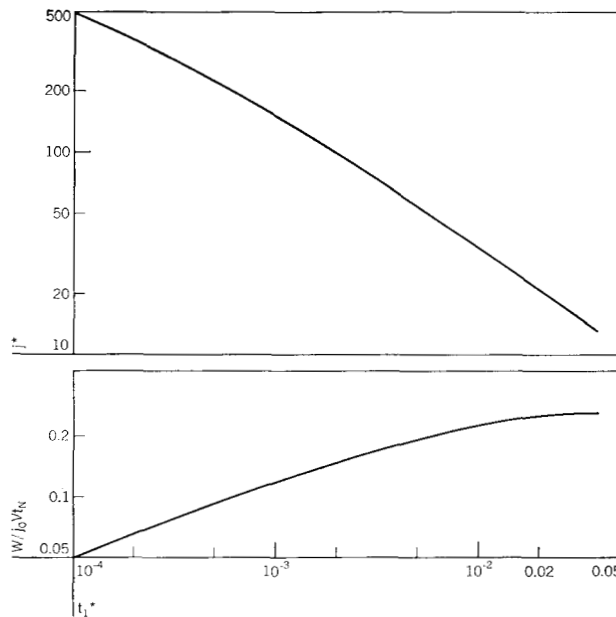


Figure 2 An example of the dependence of $j^* - \exp(j^* \sqrt{t^*})$, the integrand of Eq. (5), on time.

Figure 3 The values of j^* , the current density which maximizes the stimulated pulse power for a given pulse duration and (below) the resulting maximum value of W as a function of the pulse duration, t_1^* .



achieve maximum energy in time t_1 increases, however, by a factor of about five.

The large current densities which achieve the maximum value of W according to Fig. 3 for small pulse duration lead to a practical problem concerning series electrical resistance. The power dissipated in series resistance is proportional to the square of the current density, and, thus becomes an increasingly serious objection to the quantitative validity of the present model as pulse lengths are decreased. The heat generated in series resistance is independent of position in the model of the present section. Thus none of this heat flows, and the joule heat merely raises the temperature of the material in which it is produced. The temperature rise in the adiabatic case during a time t is

$$\Delta T = j^2 t / \sigma C. \quad (12)$$

Here σ is the electrical conductivity of the material. The ΔT of Eq. (12) must be added to that of Eq. (4). Thus, instead of the dependency in Eq. (5), the threshold current density depends on time, as follows:

$$j_t^* = \exp(j^* \sqrt{t^*} + \lambda j^{*2} t^*). \quad (13)$$

Here the reduced variables introduced in Eqs. (7) to (10) have been used. λ is a normalized measure of the electrical resistivity

$$\lambda \equiv \frac{\pi}{4} \left(\frac{\kappa T_1}{\sigma V^2} \right). \quad (14)$$

Note that j_t^* depends on j^* and t^* in the combination $j^{*2} t^*$ in Eqs. (10) and (13).

Again we want to maximize the integral of Eq. (5) and again the details are given in Appendix A. The resulting values of j^* and W are shown in Fig. 4 as functions of t_1^* and λ . The values for $\lambda = 0$ are, of course, the same as those of Fig. 3.

Now we consider the effects of letting heat flow away from a finite junction into a three-dimensional semi-infinite heat sink. In order to construct a problem with spherical symmetry, which facilitates analytical treatment, we have regarded the junction as a hemispherical heat source of radius a . It is found in this case that the temperature rise of the junction at a time t after the current is turned on is

$$\Delta T = \frac{jVa}{\kappa} [1 - e^{D/a^2} \operatorname{erfc}(Dt/a^2)^{1/2}].$$

The dependence of threshold current on time now is

$$j_t = j_0 \exp \left\{ \left[\frac{jVa}{\kappa T_1} \right] [1 - e^{D/a^2} \operatorname{erfc}(Dt/a^2)^{1/2}] \right\}. \quad (15)$$

Again, it is desirable to find that value of j which max-

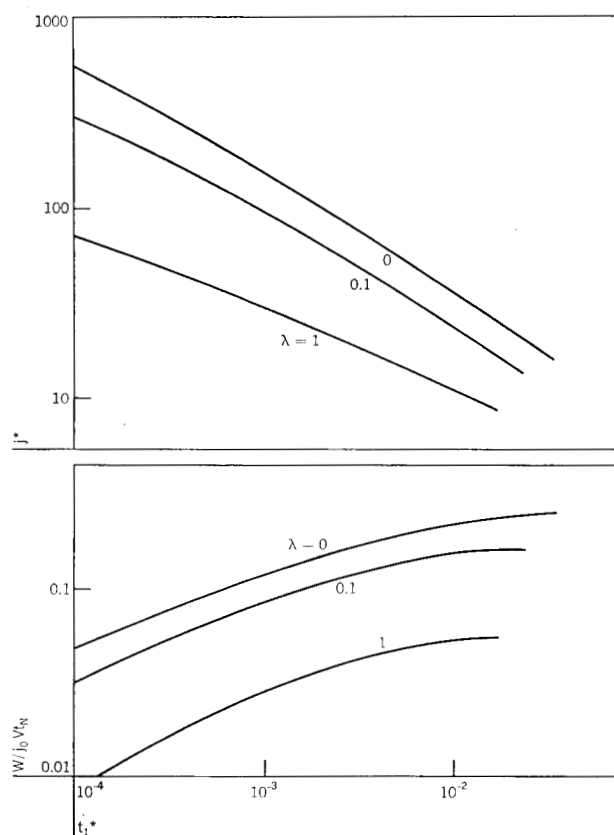


Figure 4 The quantities given in Fig. 3 when electrical resistance is taken into account for various values of the electrical resistivity parameter, λ . The curves for $\lambda = 0$ are identical with those of Fig. 3.

imizes an integral like Eq. (5). The problem is, however, too complex to be solved analytically in a useful way. There are two interesting limiting cases. One is that in which $(Dt/a^2) \ll 1$. Then Eq. (15) becomes

$$j_t = j_0 \exp \left[\frac{jVa}{\kappa T_1} \left(\frac{4Dt}{\pi} \right)^{1/2} \right], \quad (16)$$

which is identical to Eq. (6). Thus, if $Dt_1/a^2 \ll 1$, the calculations and results for the one-dimensional case are applicable. Writing $t_1 = t_1^* t_N$ and using Eq. (9) for t_N gives

$$[(j_0 Va)/(\kappa T_1)]^2 \gg t_1^* \quad (17)$$

as the condition which the material parameters must satisfy in order for the one-dimensional result to be applicable.

The physical significance of this case is clear. As seen in Fig. 1, $(Dt)^{1/2}$ is the distance to which heat penetrates in time t , and the condition that Dt/a^2 is small merely means that the penetration distance is small compared to

the linear size of the laser. It is apparent that the one-dimensional and three-dimensional cases should then lead to the same result and that this equivalence holds in the small Dt/a^2 limit for shapes other than the hemispherical one which we have assumed.

Eq. (15) can also be treated easily in the opposite limiting case, that in which Dt/a^2 becomes very large. In contrast to the one-dimensional case, the junction temperature and the threshold current density do not increase without bound in the three-dimensional case when t approaches infinity; rather, they asymptotically approach the values

$$\Delta T = jVa/\kappa, \quad (18)$$

$$j_t = j_0 \exp(jVa/\kappa T_1). \quad (19)$$

The laser will continue to operate indefinitely, or continuously, if j_t , the threshold current density of Eq. (19), is less than j , the current density. The condition that a laser can be operated continuously is thus that there is a value of j which satisfies the equation

$$j = j_0 \exp(jVa/\kappa T_1). \quad (20)$$

There is such a value of j if

$$(j_0 Va)/(\kappa T_1) < 1/e. \quad (21)$$

The present treatment of the question of continuous operation is inadequate in that it assumes a hemispherical junction, and the question will be investigated in more detail in Part 2. First, however, it is interesting to compare the regimes of the parameters which characterize the material and structure defined by the inequalities (17) and (21). Note that the parameters enter both conditions in the same combination. Depending on the value of r_1^* , there may be intermediate values of $(j_0 Va/\kappa T_1)$ for which neither approximation is valid. It has been seen, however, in connection with Fig. 3 and the discussion of the one-dimensional case, that the interesting values of r_1^* are usually no greater than 0.04 and are frequently much smaller. Thus it will often be the case that both conditions (17) and (21) are satisfied and the two regimes overlap. In other words, in many cases the problem of short pulse operation of a given laser can be treated in the one-dimensional approximation, and the potential of the same laser for continuous operation can be evaluated with the three-dimensional theory. There may be, however, intermediate cases which we have not been able to treat satisfactorily.

2. Continuous operation

Equation (18) of Part 1 illustrates that when heat flows from the injection laser into a three-dimensional heat sink the temperature of the laser approaches a constant value at long times after the current has been turned on. This

result does not depend on the particular source geometry used (hemispherical), but holds for any finite source driving heat into a three-dimensional sink. The situation in the eventual steady state is described by the concept of thermal resistance, according to which the temperature rise due to flow of heat iV is

$$\Delta T = PiV. \quad (22)$$

Here P is the thermal resistance, i is the current through the laser (or the current density, j , times the area) and iV is the rate at which heat is produced. It can be seen, for example, from Eq. (18) that $P = 1/2\pi a\kappa$ for the hemispherical junction configuration used as a model in Part 1. Here in Part 2 we use this concept of thermal resistance to discuss the thermal problems involved in continuous operation of an injection laser.

An injection laser will lase only if the current passed through it exceeds some threshold current, i_t , which is a function of the temperature of the laser. If an attempt is made to operate the laser continuously by increasing the current i through it to the threshold value, the laser is heated by the current, causing i_t to rise. The threshold i_t is, therefore, a function of the current i through the intermediary of the laser temperature. The laser will operate continuously if the current i catches up with and exceeds the threshold.

The situation is illustrated qualitatively in Fig. 5, in which i_t is plotted as a function of i . The function $i_t = i$ is also plotted. In the case for curve A the equation $i = i_t(i)$ has a solution and the laser will operate continuously. In the case of B, there is no solution; the current never catches up with the threshold current. This problem will be recognized as that treated by Mayburg.² Here we want to go into considerably more detail than that given by Mayburg.

The power dissipated in the laser is about iV , where V is the energy gap of the semiconductor divided by the electronic charge. The effects of series resistance are again neglected in the initial treatment. The dependence of the threshold current on temperature may be expressed as (see Eq. (1))

$$i_t = i_0 \exp(\Delta T/T_1). \quad (23)$$

ΔT has already been given in Eq. (22):

$$\Delta T = PiV. \quad (24)$$

Here i_0 is the threshold at temperature T . Using (23) and (24), the equation $i = i_t(i)$ becomes

$$i = \exp(PiV/T_1), \quad (25)$$

which we may rewrite in the form

$$(i/i_0) = \exp[(i/i_0)(Pi_0V/T_1)]. \quad (26)$$

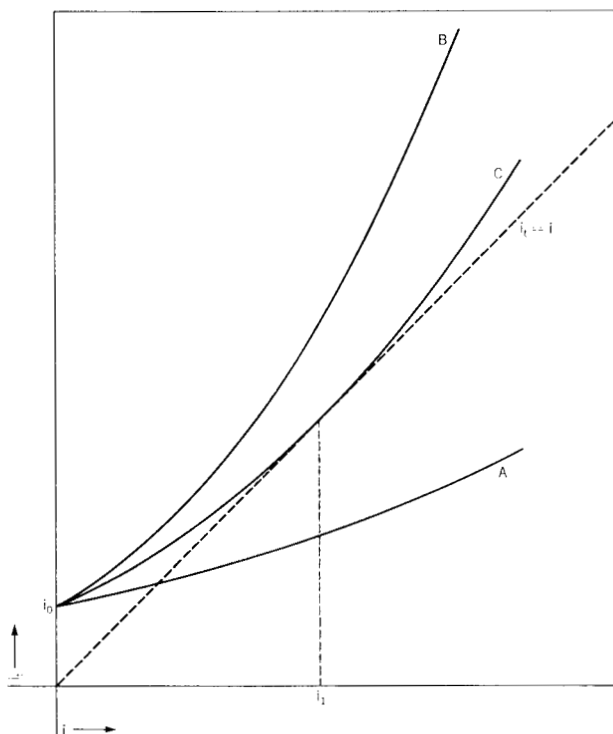


Figure 5 Examples of the way in which the threshold current depends on the current through the intermediary of the temperature in the steady state. The laser represented by curve A operates continuously at currents greater than that where curve A intersects the line $i_t = i$. On the other hand, the current never catches up with the threshold current in the laser represented by curve B, and the laser will not operate continuously. Curve C is a borderline case, and illustrates that the continuous threshold cannot be greater than i_t , which is e times i_0 .

Whether or not this equation has a real solution depends on the value of the dimensionless parameter (Pi_0V/T) . It has a solution if

$$Pi_0V/T_1 < 1/e. \quad (27)$$

Equation (27) is the condition that the injection laser can be operated continuously.

It is of interest to examine the case C of Fig. 5, which corresponds to equality in Eq. (27). The current i_1 is the maximum value to which the threshold may rise due to heating by the current and yet permit lasing to occur. i_1 is related to i_0 by $i_1 = i_0e$. The temperature has increased by an amount T_1 at current i_1 .

Now, in order to give more meaning to Eq. (27), P will be interpreted as a spreading resistance. Thus it can be written in the form

$$P = 1/2\kappa d. \quad (28)$$

Here d is a parameter with the dimensions of length and

a value of the same order of magnitude as the linear dimensions of the laser and κ is the thermal conductivity of the base through which the heat is dissipated. It is also convenient to refer the threshold current, i_0 , to current density by writing

$$i_0 = j_0wL, \quad (29)$$

where w is the width of the laser and L is its length. Combining Eqs. (27), (28), and (29) gives a condition on threshold current density

$$j_0L \leq (2d/we)(\kappa T_1/V). \quad (30)$$

It has been found that j_0L tends to be a constant for a given series of lasers, that is, for lasers made from a particular material and diffusion run.⁴ The constant j_0L is to be compared with the quantity $(\kappa T_1/V)$ multiplied by a dimensionless factor of order of magnitude unity. j_0L and $\kappa T_1/V$ are of similar magnitude in some cases of practical interest, and it is consequently worthwhile to investigate the exact value of the spreading resistance in more detail.

The spreading resistance of a rectangular shape is not known to us. As an approximation, it seems sufficiently accurate to use the spreading resistance of an ellipse with the same width-to-length ratio and the same area as the rectangle. In terms of d ,

$$\frac{1}{d} = \frac{1}{\pi^{1/2}L} K \left(1 - \frac{w^2}{L^2} \right). \quad (31)$$

Here K is the elliptic integral function whose properties are described, for example, in Jahnke-Emde.⁵ Thus Eq. (30), the condition for continuous operation, can be written

$$j_0L \leq \gamma e^{-1}(\kappa T_1/V). \quad (32)$$

The numerical factors apart from e^{-1} have been lumped into another number γ , which is a function of (w/L) , the width-to-length ratio:

$$\gamma = 2\pi^{1/2} \left[\frac{w}{L} K \left(1 - \frac{w^2}{L^2} \right) \right]^{-1}; \quad (33)$$

the function γ is shown in Fig. 6.

Now we investigate the role of dissipation in series electrical resistance in the problem of continuous operation, using a result derived in Appendix B. The temperature increase of the junction caused by the heat produced in the series resistance can be expressed as

$$\Delta T_{(\text{series } r)} = P^2 i^2 \kappa / 2\sigma \quad (34)$$

by combining Eqs. (B4), (B5), and (B10). The total temperature increase is found by adding the contributions of Eqs. (24) and (34). The equation $i = i_t(i)$, the analogue of Eq. (26), now is

$$\frac{i}{i_0} = \exp \left[\frac{i}{i_0} \frac{P i_0 V}{T_1} + \frac{1}{2} \left(\frac{i}{i_0} \right)^2 \left(\frac{P i_0 V}{T_1} \right)^2 \frac{\kappa T_1}{\sigma V^2} \right] \quad (35)$$

The electrical conductivity appears again only in the reduced form λ encountered in the pulsed case and defined by Eq. (14). Eq. (27) is now replaced by

$$(P i_0 V / T_1)^{-1} > \frac{1}{2} (1 + \sqrt{1 + 16\lambda/\pi}) \times \exp \left[\frac{1}{2} + \frac{1}{1 + \sqrt{1 + 16\lambda/\pi}} \right]. \quad (36)$$

The dependence of $P i_0 V / T_1$ on λ is shown in Fig. 7. When $\lambda = 0$ the condition for continuous operation is given by Eqs. (32) and (33) but, if $\lambda \neq 0$, the (e^{-1}) on the right hand side of Eq. (32) must be replaced by a value given by Eq. (36) and Fig. 7.

3. Application to GaAs

The most important example of the injection laser is the gallium arsenide laser; here in Part 3 the relatively formal results of the preceding parts are applied to it. The properties of the gallium arsenide laser are given in Table 1. The columns at farthest left and right list several temperatures at which the GaAs laser might be operated. Column I gives the threshold current density at the various temperatures. The threshold current density is, of course, not a constant, but varies with the material and fabrication process of the laser; the values given are typical low values, not the lowest ever attained, but those which can be achieved fairly consistently for large lasers. Column II gives the thermal conductivity, κ , of GaAs. κ is somewhat structure sensitive at the lower temperatures of the table; the values given refer to the heavily doped GaAs used in injection lasers. κ has a maximum near 20°K. Column III gives C , the specific heat per unit volume, and the ratio κ/C , the thermal diffusivity, is given in Column IV. Column V shows the values of t_N calculated from Eq. (9) using, in addition to j_0 , κ , and C which are given in the table, $V = 1.5$ Volt and $T_1 = 55^\circ\text{C}$. Column VI gives $j_0 V t_N$. Column VII gives t_0 , which is equal to $0.0394 t_N$. The quantity t_0 is the length of the pulse which gives maximum stimulated pulse energy. Column VIII gives the values of current density at which the laser must be operated to maximize the stimulated pulse energy. These values are $13.6 j_0$, as discussed in connection with Eq. (10) and Fig. 3.

The integral of Eq. (5) has the value $0.255 j_0 V t_N$ for the maximizing pulse which is characterized by the parameters of Columns I and II. The maximum light energy which can be obtained from a GaAs laser is found by multiplying $0.255 j_0 V t_N$ by an area and an efficiency factor. Large lasers tend to have low efficiency,

and the way in which the compromise between area and efficiency must be made will not be entered into here, as we are primarily concerned with thermal problems. It has, however, been considered by Lasher and Smith.¹

The amount of stimulated light energy which can be obtained in pulses shorter than t_0 can be obtained from Fig. 3. As an example, consider a 3 μsec pulse in a laser at 77°K. Since t_N is 1500 μsec , t_1^* is 0.002. The energy output per unit area is $0.14 j_0 V t_N$ instead of $0.255 j_0 V t_N$.

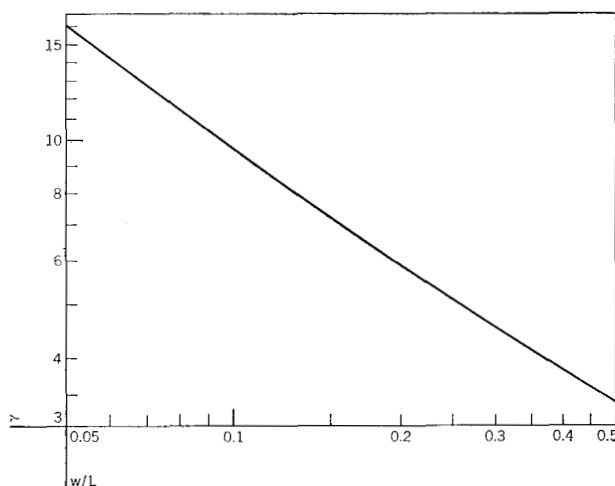


Figure 6 Values of the parameter γ of Eqs. (32) and (33).

Figure 7 The reduction in the maximum permissible value of $P i_0 V / T_1$ in the condition for continuous operation due to the effects of electrical resistance.

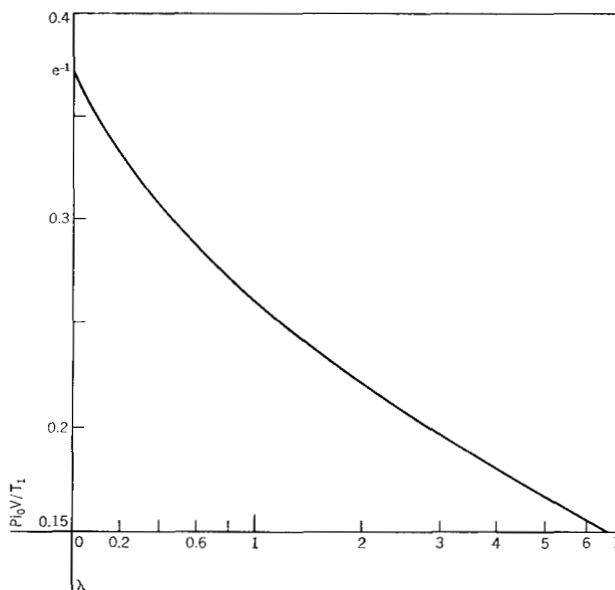


Table 1 Properties of the gallium arsenide injection laser.

I	II	III	IV	V	VI	VII	VIII	IX	X	XI		
T [°K]	j_0 [A/cm ²]	κ [W/cm deg]	C [J/cm ³ deg]	D [cm ² /sec]	t_N [μsec]	$j_0 V t_N$ [J/cm ²]	t_0 [μsec]	j [A/cm ²]	$(Dt_0)^{1/2}$ [cm]	λ (See note*)	Ohmic Degradation	T [°K]
4.2	250	3	2.1×10^{-4}	1.43×10^4	100	0.04	4	3500	0.25	0.06	0.72	4.2
20	350	10	0.021	47	1800	1	70	5000	0.057	.2	0.44	20
55	700	5	0.37	13.6	4000	4	160	10,000	0.05	.1	0.60	55
77	1000	2	0.70	3.0	1500	2	60	13,500	0.013	.04	0.82	77
300	60,000	0.5	1.70	0.30	0.24	0.02	0.01	800,000	5×10^{-6}	.01	1.00	300

* Assuming the electrical conductivity, σ , is $10^3 \text{ ohm}^{-1} \text{ cm}^{-1}$.

The energy is relatively insensitive to t_1^* ; a pulse 20 times shorter than t_0 yields half as much energy. It is also found from Fig. 3 that the current must be increased from $13.6 j_0$ to $95 j_0$ to obtain the maximum energy.

The pulse energy passes through a maximum in the range 40 to 80°K. The reason is that both κ and C are large here. At very low temperatures, although κ remains large down to the helium boiling point, C becomes small and very little energy is needed to raise the temperature of the solid. Our calculations are too pessimistic here, as they assume a temperature independent C , whereas, in fact, C increases rapidly with increasing temperature. At high temperatures C tends to become constant but κ decreases as the reciprocal of the temperature. Furthermore, the threshold current becomes large at high temperatures so that extremely short pulses must be used.

Column IX of Table I gives values of $(Dt_0)^{1/2}$, the thermal diffusion length, or distance to which heat penetrates during a maximum energy pulse. These figures are useful in determining whether the semi-infinite model and the one-dimensional model are valid in particular cases.

Thus far series resistance has been neglected in the discussion of the limits on pulsed operation of the GaAs laser. The parameter λ , the dimensionless reduced resistivity defined by Eq. (14), is given in Column X. The extra heating which comes from ohmic losses can be evaluated from the theory of Eqs. (12) to (14), Fig. 4, and Appendix A. It reduces the energy capability of the GaAs laser. The fraction by which the energy capability of the laser must be multiplied under conditions which maximize the pulse energy is given in Column XI of Table 1. The loss of energy capability of the GaAs laser due to ohmic heating amounts to a little more than a factor of two in the worst case. With shorter pulses, in which the current is higher, the ohmic effect may be larger.

Turning now to continuous operation, relevant numbers are assembled in Table 2. The same temperatures used in Table 1 are again listed at the left. As mentioned

Table 2 Parameters pertaining to continuous operation of the GaAs laser.

	I	II
T [°K]	$j_0 L$ [A/cm]	$\kappa T_1/V$ [amp/cm]
4.2	10	110
20	14	370
55	28	180
77	40	75
300	2000	20

previously, it has been found that $j_0 L$, the threshold current density times the length of the laser, is approximately constant in small lasers, which are most suitable for continuous operation.⁴ Values of $j_0 L$ are given in Column I. Values of $(\kappa T_1/V)$, the quantity with which $j_0 L$ has to be compared according to Eq. (30), are given in Column II. It is seen that $j_0 L$ is indeed small compared to $(\kappa T_1/V)$ at temperatures less than 55°K and, therefore, that continuous operation of GaAs injection lasers below this temperature should be easy, a result which is confirmed by experience.

In the case of operation in a bath of boiling liquid nitrogen, at a temperature of 77°K, $j_0 L$ and $(\kappa T_1/V)$ are not greatly different. The question of the feasibility of continuous operation here reduces to one of detail. To explore this detail Fig. 8 has been constructed from Eqs. (32) and (33) and the experimental measurements of Pilkuhn, Rupprecht, and Blum.⁴ The content of this Figure is that $j_0 L$ must lie below the solid lines drawn from various values of w for continuous operation to be possible. The dotted line shows $j_0 L$ as measured by Pilkuhn et al. It appears that the experimental results lie well below the limit set by Eq. (32) for small lasers, so that

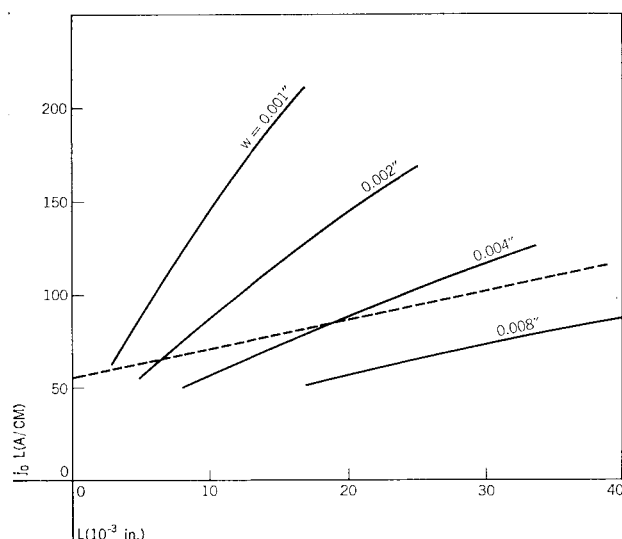


Figure 8 Application of Eq. (32) to the problem of continuous operation of the gallium arsenide injection laser at 77°K. The laser will operate continuously if j_0L lies below the value given by the solid lines for the particular width in question. The values of j_0L found by Pilkuhn, Rupprecht, and Blum are shown by the dotted line.

continuous operation should be easy if a proper heat sink is provided.

The situation is quite different at 300°K, probably the most interesting temperature; here j_0L is two orders of magnitude larger than $(\kappa T_1/V)$. In principle, of course, one might hope to make up even this large difference with the factor γ of Eqs. (32) and (33) and Fig. 6. In fact, it turns out that the length-to-width ratio required to make γ sufficiently large is 500, which seems to be outside the scope of present day injection laser technology.

The effect of series ohmic resistance on continuous operation of the GaAs injection laser can be evaluated from Fig. 7, which shows the numbers by which the e^{-1} of Eqs. (32) and (33) must be replaced when series resistance is taken into account. The values of the reduced resistivity, λ , have already been given in Table 1. The effect of series resistance is always small; even in the worst case, around 55°K, the numerical factor e^{-1} is changed only by 10%.

Practical lasers have much more complicated structures than the idealizations of Parts 1 and 2. Perhaps the most important difference is that the junction is not really attached directly to the heat sink. A certain amount of series thermal resistance intervenes between the junction and the spreading resistance of the heat sink. This series thermal resistance is the kind of resistance considered in the calculation of Mayburg.² A comparison of the series thermal resistance with the spreading resistance described by Eq. (28) and the subsequent theory shows that the

series resistance will be small compared to the spreading resistance if the height of the junction above the heat sink is small compared to w , the width of the laser. It is not too difficult, however, to extend the application of Eq. (27) to cases in which P is the sum of series and spreading resistance.

Another complication which arises in practice is that heat may be allowed to flow away from the junction in both directions, so that there are two thermal conductances in parallel. If these two parallel paths are identical, they introduce no additional complication in the analytic treatment, and can be described by modifying the one-sided theory in an obvious way. Usually, however, at least the series thermal resistance will be different on the two sides. If one of these is much larger than the other, the one-sided theory may become applicable.

Exact treatment of problems involving parallel and series combinations of thermal resistance are possible by the methods introduced in this paper, although the formulas involved will frequently become considerably more complicated.

Conclusions

The present paper solves the thermal problems associated with short pulse and steady state operation of an idealized model of an injection laser. The physical process which limits the light that can be produced by the injection laser is the heating produced by the passage of current which, since the threshold current depends on temperature (Eq. (1)), raises the threshold. Stimulated emission is produced only by the current in excess of the threshold current, however. The race between the current and the threshold current, which, through the temperature, depends on the current, was illustrated in Fig. 5; if the current catches up with the threshold current, stimulated emission begins (Curve A of Fig. 5) and, as the current is increased, the current in excess of the threshold produces stimulated emission. The stimulated emission passes through a maximum as the current continues to increase and eventually the threshold current again exceeds the current and stimulated emission stops.

The condition that the laser operate continuously was given in terms of characteristic parameters of the laser structure in Eqs. (21) and (27)–(30). These conditions are easy to satisfy with common gallium arsenide lasers at temperatures somewhat below 77°K. They are very difficult to satisfy at temperatures above 77°K. The junction temperature may be as much as T_1 (usually in the range 40°–80° in GaAs) higher than the ambient temperature at the point at which lasing begins (Curve C of Fig. 5).

Even if the laser does not operate continuously (Curve B of Fig. 5), useful stimulated light may be obtained in short pulses. If a current density greater than j_0 is suddenly

switched through the laser, stimulated light will be produced until the heat produced raises the threshold current up to the value of the applied current. There is a value for the applied current which maximizes the amount of stimulated light obtained in a pulse. If the applied current is near the threshold current the excess current which produces stimulation emission is small, and, furthermore, only a small amount of heating can destroy the lasing action. If the applied current becomes very large the stimulated light output again becomes small because the time needed for the heat to destroy the lasing action is nearly proportional to the inverse square power of the current (Eq. (6)). The problem of finding the current which maximizes the amount of simulated light in a pulse is solved in Part 1 and Appendix A. It turns out that the optimum current density is $13.6 j_0$ and the stimulated light output is $0.25 j_0 V t_N$ times an efficiency factor and an area. Here t_N is a characteristic time of the laser defined in Eq. (9).

The theory was extended to take into account the heating caused by the passage of the current through the electrical resistance of the laser material. The heating in series electrical resistance has little effect on the continuously operating GaAs laser, but may seriously degrade performance at the high currents used in pulsed lasers.

Appendix A: Maximum of the stimulated power integral

This Appendix considers the problem of finding that value of current which maximizes the stimulated power integral W , defined by Eq. (5) and Eq. (10) or Eq. (13). Eq. (5) is

$$W = j_0 V t_N \int_0^{t_1^*} (j^* - j_i^*) dt^*, \quad (A1)$$

in terms of the variables defined in Eqs. (7) to (11). The maximization problem involves solving:

$$\begin{aligned} 0 &= (j_0 V t_N)^{-1} dW/dj^* \\ &= (j^* - j_i^*(t_1^*)) (dt_1^*/dj^*) \\ &\quad + \int_0^{t_1^*} (1 - \partial j_i^*/\partial j^*) dt^*. \end{aligned} \quad (A2)$$

If the pulse length is fixed, t_1^* is a constant, and the first term on the right hand side of Eq. (A2) is zero. Recall that j_i^* is a function of $j^* t^*$ in Eqs. (10) and (13). Thus

$$\partial j_i^*/\partial j^* = 2(t^*/j^*)(\partial j_i^*/\partial t^*). \quad (A3)$$

Substituting Eq. (A3) into Eq. (A2) and performing an integration by parts reduces Eq. (A2) to

$$j^* = 2 \left[j_i^*(t_1^*) - \frac{1}{t_1^*} \int_0^{t_1^*} j_i^* dt^* \right], \quad (A4)$$

an implicit equation for j^* because j_i^* depends on j^* , as in Eq. (6). When j^* has the value given by Eq. (A4), W is

$$W = j_0 V t_N t_1^* [(3/2)j^* - j_i^*(t_1^*)]. \quad (A5)$$

The results given by Eqs. (A4) and (A5) only have meaning when $j_i^*(t_1^*)$ is less than j^* , that is, the threshold current at the end of the pulse is still less than the current. Starting with small values of t_1^* , W as given by Eq. (A5) is an increasing function of t_1^* until t_1^* is such that $j_i^*(t_1^*) = j^*$. Then the current which maximizes W is such that the laser turns off due to heating just at the end of the pulse. The value of t_1^* at which this occurs is t_0^* . Thus t_0^* is the value of t_1^* for which both Eq. (A4) and

$$j_i^*(t_1^*) = j^* \quad (A6)$$

are satisfied. W cannot be increased by making t_1^* longer than t_0^* .

Because of Eq. (A6) the first term on the right hand side of Eq. (A2) vanishes also if, instead of fixing t_1^* , it is allowed to be determined by the time at which the laser turns itself off due to heating. Thus the value of W corresponding to t_0^* is the maximum pulse energy which can be obtained under any conditions.

If the integral in Eq. (A4) is evaluated using Eq. (10), Eq. (A4) becomes

$$\begin{aligned} j^* &= [2 - 4(j^{*2} t_1^*)^{-1/2} + 4(j^{*2} t_1^*)^{-1}] \\ &\quad \times \exp(j^{*2} t_1^*)^{1/2} - 4(j^{*2} t_1^*)^{-1}. \end{aligned} \quad (A7)$$

If, instead, Eq. (13) is used to evaluate the integral, Eq. (A4) becomes

$$\begin{aligned} j^* &= 2 \left[1 - \frac{1}{\lambda j^{*2} t_1^*} + \frac{1}{\lambda^{3/2} j^{*2} t_1^*} \right. \\ &\quad \times I \left(\frac{1}{2\lambda^{1/2}} + \lambda^{1/2} (j^{*2} t_1^*)^{1/2} \right) \Big] \\ &\quad \times \exp[(j^{*2} t_1^*)^{1/2} + \lambda j^{*2} t_1^*] \\ &\quad + \frac{2}{\lambda j^{*2} t_1^*} - \frac{2}{\lambda^{3/2} j^{*2} t_1^*} I \left(\frac{1}{2\lambda^{1/2}} \right). \end{aligned} \quad (A8)$$

Here $I(z)$ is the function^{3,6}

$$I(z) = e^{-z^2} \int_0^z e^{x^2} dx. \quad (A9)$$

Equations (A7) and (A8) were used to construct Figs. 3 and 4.

Appendix B. The thermal effect of series resistance

In this Appendix we wish to find the temperature rise at a surface due to the ohmic heat produced by a current as it flows away from the surface through a spreading resistance. Consider that space is divided into two semi-infinite parts by a surface Q , the right hand part being the medium under consideration. Q has two parts, a bounded part S through which electrical current is injected into the medium, and the infinite remaining part S_2

which is electrically and thermally insulating. The electrical spreading resistance is found by solving the following equations which define the electric potential, ϕ :

First, Laplace's equation,

$$\nabla^2 \phi = 0; \quad (\text{B1})$$

Second, the condition that S is an equipotential,

$$\phi = \phi_1 \text{ on } S; \quad (\text{B2})$$

Third, the condition that no current flows across S_2 ,

$$\mathbf{n} \cdot \nabla \phi = 0 \text{ on } S_2. \quad (\text{B3})$$

In Eq. (B3) $\mathbf{n}$ is a vector normal to the surface Q , and ϕ vanishes at infinity.

The solution of the electrical potential problem permits the definition of an electrical spreading resistance, R , as follows:

$$R = \phi_1 / i = \phi_1 / \int_S \sigma \nabla \phi \, dS. \quad (\text{B4})$$

An entirely analogous thermal problem, in which T replaces ϕ and κ replaces σ , can be solved. A thermal spreading resistance is thus obtained which has the value

$$P = (\sigma / \kappa) R. \quad (\text{B5})$$

Turning now to the actual thermal problem, the divergence of the heat current, J , is equal to the rate of production of electrical heat

$$\nabla \cdot J = \sigma (\nabla \phi)^2. \quad (\text{B6})$$

Writing $J = -\kappa \nabla T$ and $(\nabla \phi)^2 = \nabla \cdot (\phi \nabla \phi) = \frac{1}{2} \nabla^2 (\phi^2)$, Eq. (B6) becomes

$$-\kappa \nabla^2 T = \frac{1}{2} \sigma \nabla^2 (\phi^2). \quad (\text{B7})$$

The desired solution of Eq. (B7) is

$$T = (\sigma / 2\kappa) (2\phi_1 \phi - \phi^2). \quad (\text{B8})$$

The T of Eq. (B8) satisfies Eq. (B7) and the following boundary condition:

$$\Delta T = 0 \quad (J = 0) \text{ on } S.$$

This condition insures that the heat flows away through the medium, the heat sink, and not the junction

$$\mathbf{n} \cdot \nabla T = 0 \quad (\mathbf{n} \cdot J = 0) \text{ on } S_2,$$

as required by the statement that S_2 is thermally insulating.

$$T = 0 \text{ at infinity.}$$

This is a definition of the zero of the temperature scale. With this definition, T is the increase in temperature over the bath temperature.

It is seen from Eq. (8) that the temperature increase at the junction, where $\phi = \phi_1$, is

$$\Delta T = (\sigma / 2\kappa) \phi_1^2. \quad (\text{B9})$$

By recalling Eq. (B5), Eq. (B9) can be written

$$\Delta T = \frac{1}{2} P (\phi_1^2 / R). \quad (\text{B10})$$

However, ϕ_1^2 / R is the electrical power dissipated. The temperature increase is that which would be produced by the flow of half of the electrical power through the thermal spreading resistance.

Appendix C: A tabulation of notation

a	Radius of hemispherical contact to heat sink
C	Specific heat per unit volume
D	Thermal diffusivity, κ / C
i	Electrical current
i_0	Threshold current of a laser at the bath temperature, T
i_t	Threshold current of a laser at actual junction temperature
i_1	A current defined by Fig. 5
j	Electrical current density
j_0	Threshold current density at bath temperature
j_t	Threshold current density at actual junction temperature
j^*	Reduced current density, j / j_0
j_t^*	Reduced threshold current density, j_t / j_0
K	Elliptic integral
L	Length of a laser
R	Electrical resistance
S	The junction surface
t	Time
t_0	Time at which lasing stops due to heating of the laser
t_1	Duration of current pulse
t_N	A natural time which occurs in thermal problems of the injection laser and is defined by Eq. (9)
t^*	t / t_N
t_0^*	t_0 / t_N
t_1^*	t_1 / t_N
T	Bath temperature
T_1	A parameter which characterizes the temperature dependence of threshold current, as in Eq. (1)
ΔT	The amount by which the temperature of the junction exceeds the bath temperature

V	Voltage
w	Width of a laser
W	Integrated electrical power above threshold, see Eq. (5)
$W_{\max}$	The maximum value which W can have, Eq. (11)
γ	Numerical factor in the criterion for continuous operation, Eq. (33)
κ	Thermal conductivity
λ	Normalized electrical resistivity, Eq. (14)
P	Thermal resistance
σ	Electrical conductivity

References

1. G. J. Lasher and W. V. Smith, *IBM Journal* 8, 532 (1964).
2. S. Mayburg, *J. Appl. Phys.* 34, 3417 (1963).
3. *Handbook of Mathematical Functions*, U. S. National Bureau of Standards, Washington (1964).
4. M. Pilkuhn, H. Rupprecht, and S. Blum, *Solid State Electronics* 7, 905 (1964).
5. E. Jahnke and F. Emde, *Tables of Functions*, Dover, New York, 1945.
6. W. L. Miller and A. R. Gordon, *J. Phys. Chem.* 35, 2785 (1931).

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